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**MULTIOBJECTIVE OPTIMIZATION – POSSIBILITY FOR PRODUCTION
IMPROVEMENT**

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Abstract: *In the previous research strong correlation was discovered between the features of the product drawing and production time, which has resulted with 8 regression equations. They were realized using stepwise multiple linear regressions. Since the optimization of these regression equations did not fully define the most frequent requirements, multiobjective optimization was applied. The applied criteria included: minimum production time, maximum work costs/total costs ratio for a group of workpieces. The group was created using specific classifiers that defined similar workpieces. A STEP model with seven decision variables within a group was applied, and the groups with a high index of determination were selected. Independent values that maximize the work costs/total costs ratio and minimize production times were determined. The obtained regression equations of time production parts and work costs/total costs ratio are included in the objective functions to reduce production time and increasing, work costs/total costs at the same time. The values of decision variables that minimize production time and maximize work costs/total costs ratio were determined. As the solution of the described problem, multicriteria interactive STEP method was applied.*

Key words: *stepwise multiple linear regression, multiobjective optimization, STEP method*

1. INTRODUCTION

Our numerous experiences and experience of others as well, and following of economic trends in Croatia and wider have motivated us to start research in this area. Since a considerable number of research works and papers are dealing with optimization of technological parameters, we have decided to focus our attention on the relationship between product features (geometry, complexity, quantity,...) and production times and costs [1,2,3,4]. It has been proved that it is possible to make estimation of production time applying classification, group technology, stepwise multiple linear regression as the basis for accepting or rejecting of orders, based on 2D [1, 2] drawings, and the set basis for automatic retrieval of features from the background of 3D objects (CAD: Pro/E, CATIA) and their transfer to regression models [5]. Of course, certain constraints have been set: application of standardized production times from technical documentation or estimations made using CAM software (CATIA, PRO/E, CamWorks); type of production equipment/technological documentation determines whether it will be single- or low-batch production. Initial steps have been taken regarding medium-batch, large-batch or mass production.

It has been assumed (relying on experience) that small companies (SMEs) in Croatia make decision about acceptance of production (based on customer's design

solution of the product, delivery deadlines and manufacturing costs imposed by the customer - PICOS concept) on the basis of free intuitive assessment due to the lack of time and experts. This often results in wrong estimates.

If the optimization of regression curves is to be applied (independent variables - product features, dependent variable - production time), it is hard to explain what it would mean for the minimum or maximum production time for a given group of products. The minimum production time could mean a higher productivity, but we do not know about the profit. The maximum production time could suggest that a higher occupancy of capacities may mean higher earnings, although it may not be so. This dual meaning has led us to introduce multiple objective optimizations for a new class of variables that differently classify our products. A response variable (dependent variable) can assume several meanings: maximum profit per product, minimum delivery time (related to production time, and also to organizational waste of time, production balancing...), ratio of the production cost and the costs of product materials, ratio of the production cost and the ultimate production cost. Thus, the problem-solving approach has become more complex, and is no longer a mere result of intuition and heuristics, but more exact assessment of 'common' optimum for more set criteria.

2. THEORETICAL BACKGROUND

TABLE 1. Minimum and maximum values of selected variables

PRODUCT TYPE - 41113									
min	2.90	0.100	1.00	11.21	0.22	0.0132	0.001	6.00	0.92
max	100.00	0.400	5.00	19.63	12.50	0.3972	0.820	33.00	1.00
variable	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	Z ₁	Z ₂
Variable description	Workpiece outer diameter	Narrowest tolerance of measures	Scale of the drawing	Material mass/strength ratio	Wall thickness/length ratio	Product surface area	Material mass	Production time	Ratio of Work costs/total costs
unit of measure	mm	mm	number	number	number	10 ⁴ mm ²	kg	h/100	number

Used technological documentation for conventional machining tools (420 positions) is from INAS Company, a successful producer of machine tools in Croatia. The main grouping criteria were the features (geometrical, tolerance, hardness) from technical drawings and for each workpiece production time was used (technological and auxiliary time).

It was found that the optimization of regression equations, in order to obtain minimum or maximum production times was insufficient with respect to the needs in real production. Thus, the aim was to obtain, by considering a series of regression equations, the optimum for multiobjective optimization (minimal production time, labor cost/material cost ratio or labor cost/total cost ratio for the selected group of products. As multiobjective optimization requires the same variables (x₁, x₇), it was necessary to make new grouping of the basic set (302 workpieces) using new classifiers. New classifiers were defined W (1-5), based on 5 basic features:

W1-material: 1(polymers)-5(alloy steel), W2-shape: 1(rotational)-5(complex), W3- max. workpiece dimension: 1(mini V<120mm)-5(V>2000 mm), W4-complexity, BA – number of dimension lines: 1(very simple BA≤5)-5(very complex BA>75), W5-treatment complexity: 1(very rough)-5(very fine). The conditions were defined based on the range of data about the number of dimension lines on the considered sample

of 415 elements. A classifier that is being developed is based on 5 basic workpiece features. For the purpose of the research, a group of workpieces (W1-W5) 41113 was selected for further analysis. The code 41113 means: steel – rotational – small – very simple – commonly complex - workpieces. From the available database, the minimum and maximum values for independent variables, and dependent variable (Z₁-production time), and derived variable Z₂ was taken (Table 1.).

Two regression equations, Z₁ (production time) and Z₂ (labor cost/total cost ratio), were selected. For them multiobjective optimization was also performed. In order to use the same types of variables, new grouping was made using specifically adjusted classifiers. *Workpiece classification according to the criterion of complexity* was done semi-automatically by setting conditions on certain features of drawings (basic roughness, the finest roughness requirement, the narrowest tolerance of measures, the narrowest tolerance of shape or position (geometry), number of all roughness and geometry requirements in the drawing. Each of these 6 criteria based on its specific conditions is assigned a value ranging from 1 to 5. The obtained result (Table 2.) is rounded to integer (e.g. 3.49 is W=3, and 3.51 is W=4), and this integer (in the range from 1 to 5) becomes complexity criterion coefficient (the fifth digit in the code).

TABLE 2. Results of stepwise multiple linear regression

Regression Statistics	Dependent variable -production time Z ₁	Regression Statistics	Dependent variable- work costs/ultimate costs ratio Z ₂
Multiple R	0.92212166	Multiple R	0.99207
R Square	0.85030835	R Square	0.984202
Adjusted R Square	0.78481826	Adjusted R Square	0.977291
Standard Error	4.09742037	Standard Error	0.002725
Observations	24.0	Observations	24.0
Z ₁	<i>Coefficients</i>	Z ₂	<i>Coefficients</i>
Intercept	-13.490042	Intercept	0.990439
X Variable 1	0.86652065	X Variable 1	0.000238
X Variable 2	-0.1993556	X Variable 2	-0.0039

X Variable 3	0.75343156	X Variable 3	0.00046
X Variable 4	1.41593567	X Variable 4	0.000794
X Variable 5	-1.8669075	X Variable 5	-0.00107
X Variable 6	4.83640676	X Variable 6	-0.04466
X Variable 7	-51.274031	X Variable 7	-0.08551

3. THE MULTIOBJECTIVE MODEL

The general multiobjective optimization problem with n decision variables, m constraints and p objectives is [6]:

$$\text{maximize } Z(x_1, x_2, \dots, x_n) = [Z_1(x_1, x_2, \dots, x_n), Z_2(x_1, x_2, \dots, x_n), \dots, Z_p(x_1, x_2, \dots, x_n)] \quad (1)$$

$$\text{s.t. } g_i(x_1, x_2, \dots, x_n) \leq 0, \quad i = 1, 2, \dots, m \\ x_j \geq 0, \quad j = 1, 2, \dots, n \quad (2)$$

where $Z(x_1, x_2, \dots, x_n)$ is the multiobjective objective function and $Z_1(\cdot)$, $Z_2(\cdot)$, $Z_p(\cdot)$, are the p individual objective functions. The step method [7] is based on a geometric notion of best, i.e., the minimum distance from an ideal solution, with modifications of this criterion derived from a decision maker's (DM) reactions to a generated solution. The method begins with the construction of a payoff table. The table is found by optimizing each of the p objectives individually, where the solution to the k th such individual optimization, called x^k , gives by definition the maximum value for the k th objective, which is called M_k (i.e., $Z_k(x^k) = M_k$). The values of the other $p - 1$ objectives implied by x^k are shown in the k th row of the payoff table. The payoff table is used to develop weights on the distance of a solution from the ideal solution. The step method employs the ideal solution, which has components M_k for $k = 1, 2, \dots, p$. The ideal solution is generally infeasible. The λ , metric is used to measure distance from the ideal solution. The distance is scaled by a weight based on the range of objective Z_k and the feasible region is allowed to change at each iteration of the algorithm. The basic problem in the step method is:

$$\text{Min } \lambda \quad (3)$$

$$\Pi_k [M_k - Z_k(x)] - \lambda \leq 0, \quad k = 1, 2, \dots, p \quad (4)$$

$$x \in F_d^i, \quad \lambda \geq 0 \quad (5)$$

where F_d^i is the feasible region at the i th iteration and λ is used to indicate that the original metric has been modified. Initially, $F_d^0 = F_d$; i.e., at the start of the algorithm the original feasible region is used in Eq.5 The weights π_k in Eq.4 are defined as:

$$\Pi_k = \frac{\alpha_k}{\sum_{i=1}^p \alpha_i} \quad (6)$$

$$\alpha_k = \frac{M_k - n_k}{M_k} \left[\sum_{j=1}^n (c_j^k)^2 \right]^{-\frac{1}{2}} \quad (7)$$

where n_k is the minimum value for the k th objective; i.e. it is the smallest number in the k th column of the payoff table. The c_j^k are objective function coefficients, where it is assumed that each objective is linear.

4. RESULTS

On the basis of considerations of regression functions in previous sections, the problem of multiobjective optimization with minimization of the objective functions Z_1 and Z_2 with related constraints (Eq.8 to Eq.10) is defined.

$$\begin{aligned} \text{Min } Z_1 = & -13.49004192 + 0.866520652 * x_1 \\ & 0.199355601 * x_2 + 0.753431562 * x_3 + 1.415935668 * x_4 - \\ & 1.866907529 * x_5 + 4.836406757 * x_6 - \\ & 51.27403107 * x_7 \quad (8) \\ \text{Min } Z_2 = & -0.990438731 - \\ & 0.000238475 * x_1 + 0.003897645 * x_2 - 0.00045981 * x_3 - \\ & 0.000794225 * x_4 + \end{aligned}$$

$$\begin{aligned} & 0.0010738 * x_5 + 0.044664232 * x_6 + 0.085514412 * x_7 \quad (9) \\ & x_1 \leq 100; \quad x_2 \leq 0.4; \quad x_3 \leq 5.0; \quad x_4 \leq 19.63; \quad x_5 \leq 12.50; \quad x_6 \leq \\ & 0.3972; \quad x_7 \leq 0.820 \quad (10) \end{aligned}$$

In Eq.12 and Eq.13 Z_1 represents variable T , and Z_2 variable TU/TR . It should be mentioned that for the needs of consistency of the objective functions Z_1 and Z_2 , for the objective function Z_2 (Eq.9) the signs of the coefficients of variables and of the free member have been changed. The values of objective functions Z_1 and Z_2 in the extreme points of the set of possible solutions (feasible region) are given in Table 3. It is visible from the table that there is no common set of points ($x_1 \dots x_7$) where both functions Z_1 and Z_2 have extreme (maximum) values, and thus the need for optimization of the given problem is justified.

In accordance with Eq.6 and Eq.7 coefficients of equation Eq.4 are calculated as follow: $\alpha_1 = 0.0197$, $\alpha_2 = 10.1974$, $\Pi_1 = 0.0019$, $\Pi_2 = 0.9981$. The results of the first compromise solution given in Table 4. Since in the given problem there are two objective functions, it is necessary to make calculation of the second compromise solution. It has been decided that the previous value for $M_1 = 73.1620$ is to be reduced for the value of 33.1620, and thus the new value for $M_1 = 40$.

In accordance with Eq.6 and Eq.7 coefficients of equation Eq.4 are calculated as follow: $\alpha_1 = 0.0199$,

$\alpha_2=10.1974$, $\Pi_1=0.0019$, $\Pi_2=0.9981$. The results of the second compromise solution given in Table 5.

5. CONCLUSION

The paper presents research on the development of a model for the estimation of production time for unit production or medium size batch production. As a result, eight regression equations were obtained. They show estimation of the production time as a function of geometrical and technological characteristics of a homogeneous group of products that were grouped using logical operators. Using specifically developed 5 classifiers at 5 levels, on the sample taken from the real production a homogenous group was formed which resulted in a regression equation showing dependence between production time (Z_1) and 7 independent variables (x_1, \dots, x_7). After that, the dependence between the work costs/total costs ratio (Z_2) and independent variables (x_1, \dots, x_7) is shown in another regression equation. The

optimization part of the work considers the possibility of application of standard STEP method as multiobjective optimization approach in optimization of production problems, where the objective functions are obtained by regression model. The results obtained by application of STEP method indicate that its application is possible in the optimization of decision variables of the given objective functions. It is evident that the results of both objective functions are within the statistical range, i.e. $\text{Min } Z_1(x_1, x_7) = 19.0013$ and $\text{Max } Z_2(x_1, x_7) = 0.9915$, and thus it is not necessary to introduce a new payoff table to find a new compromise (feasible) solution. The following can be concluded: it is cost-effective to manufacture products with minimum outside diameter (x_1), maximum (wider range) tolerance (x_2), maximum scale (x_3), maximum strength/mass ratio (x_4), minimum of wall thickness/length ratio (x_5), maximum product surface area (x_6) and minimum mass of material (x_7).

TABLE 3. Values of the decision variables and the objective functions

Extreme point	Decision variables							Objective functions	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$Z_1(x_1, \dots, x_7)$	$Z_2(x_1, \dots, x_7)$
A	100	0	0	0	0	0	0	73.1620	-1.0143
B	0	0.4	0	0	0	0	0	-13.5698	-0.9889
C	0	0	5	0	0	0	0	-9.7229	-0.9927
D	0	0	0	19.63	0	0	0	14.3048	-1.0060
E	0	0	0	0	12.50	0	0	-36.8264	-0.9770
F	0	0	0	0	0	0.3972	0	-11.5690	-0.9727
G	0	0	0	0	0	0	0.820	-55.5347	-0.9203

TABLE 4. Results of the first compromise solution

$x_1=100$; $x_2=0.4$; $x_3=1.0$; $x_4=12.0428$; $x_5=12.5$; $x_6=0.3962$; $x_7=9999998E-4$; $\lambda=7.128304E-2$
Min $Z_1(x_1, \dots, x_7) = 69.4161$; Min $Z_2(x_1, \dots, x_7) = -0.9915$; Max $Z_2(x_1, \dots, x_7) = 0.9915$

TABLE 5. Results of the second compromise solution

$x_1= 3.37147$; $x_2= 0.3711865$; $x_3= 4.553035$; $x_4= 18.92068$; $x_5= 0.2269908$; $x_6= 0.2826709$;
 $x_7= 2.965111E-2$; $\lambda = 7.682257E-2$
Min $Z_1(x_1, \dots, x_7) = 19.0013$; Min $Z_2(x_1, \dots, x_7) = -0.9915$; Max $Z_2(x_1, \dots, x_7) = 0.9915$

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